Efficient Mechanisms with Small Subsidies

(Extended Abstract)

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ABSTRACT

We address the problem of designing efficient mechanisms that never yield revenue, instead requiring small *subsidies*. Such mechanisms will be pertinent for settings in which taxing agents is undesirable or impractical—imagine, e.g., a government or private philanthropist that seeks to make a minimal monetary contribution that will allow a group of individuals to reach an efficient decision without being stripped of any of the surplus. Our approach is a close analog of [1], where structure in agent valuations is used to arrive at agent-independent revenue lower bounds for the VCG mechanism that form the basis for redistributing VCG revenue back to the agents without distorting incentives; in the current paper we use valuation structure to obtain revenue *upper* bounds that form the basis for a second stage of redistribution, returning the revenue of the redistribution mechanism of [1] such that revenue is non-positive but still close to zero. The mechanism we propose is applicable to arbitrary decision problems, always achieving dominant strategy efficiency, ex post individual rationality, and no-revenue. In single-item allocation settings it is asymptotically strongly budget-balanced as the population size grows; we show empirically that for standard distributions over valuations it requires subsidies that are less than 5% of social value, in expectation, for groups of more than 5 agents.

Categories and Subject Descriptors

J.4 [**Social and Behavioral Sciences**]: Economics

General Terms

Economics, Theory

Keywords

Mechanism design, redistribution mechanisms, subsidies

1. INTRODUCTION

Imagine a scenario in which a decision is to be made that will yield varying amounts of value for individuals in a group. Individuals are selfish and hold private valuation information, so there is a problem of incentives. The attitudes of the individuals and/or the social planner are such that any decision-making scheme is deemed unacceptable if it may result in the group *not* enjoying the full value potentially attainable in the setting, either by selection of a non social-welfare maximizing outcome or by requirement of aggregate monetary transfers made to an entity outside of the group. Moreover, to ensure participation no agent should be made worse off for having participated. We know that no strongly budget-balanced, interim individual rational, and dominant strategy efficient mechanism exists, even in very simple settings (see [2] and [3]); thus, given the constraints outlined above, any "acceptable" mechanism will run a deficit. We propose a mechanism geared towards minimizing this deficit.

We build on the work of Cavallo [1] who proposes a *redistribution mechanism* (RM) for returning VCG revenue to the agents. Letting n denote the number of agents, RM implements VCG and additionally pays each agent a $1/n$ share of the minimum revenue that could result under VCG given the agent's typespace and the types reported by the other agents, taken over all possible reports by the agent. The core idea of the current paper is to add a second stage of redistribution that ensures no-revenue while still coming close to perfect budget-balance.

2. MAIN RESULTS

We propose the NR mechanism, which can be described simply and intuitively as follows (we stick to prose here to avoid having to introduce notation).

DEFINITION 1 (THE NR MECHANISM). *A social-welfare maximizing outcome is chosen and each agent is: 1) charged an amount equal to the negative externality he exerts on other agents (VCG payment); 2) payed a $1/n$ share of the minimum total revenue that could result from (1), considering all possible reports by the agent (RM redistribution); and 3) payed a $1/n$ share of the maximum revenue that could result from steps (1) and (2), considering all possible reports by the agent.*

THEOREM 1. *For all typespaces the NR mechanism is truthful and efficient in dominant strategies and no-revenue; it is ex post individual rational for any typespace in which the no negative externalities condition[1] holds.*

Also, the required subsidy never exceeds social welfare, and we can achieve much better bounds for specific settings.

[1] *No negative externalities* holds if, $\forall i \in I, \theta \in \Theta$, $v_i(\theta_i, f^*(\theta_{-i})) \geq 0$, i.e., if no agent ever obtains negative value for an outcome that is optimal for the other agents.

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2.1 Single-item allocation

The mechanism is elegant and simple, conceptually and computationally, in settings where a single resource is to be allocated. Here the preferences of each agent i can be fully expressed with just a single number v_i, the value he obtains from being allocated the item. Given the context of a vector $v = (v_1, \ldots, v_n)$ of agent values, we use notation v^k to denote the k^{th} highest value and v^k_{-i} for the k^{th} highest among agents other than some i (to be clear: the k does not represent an exponent). The NR mechanism here reduces to the following form:

Definition 2. (NR mechanism in single-item allocation settings) *Given bids $\hat{v} = (\hat{v}_1, \ldots, \hat{v}_n)$, the item is allocated to the highest bidder k (with ties broken arbitrarily) and the following transfer payments are made:*

$$T_k(\hat{v}) = -\hat{v}^1_{-k} + \frac{\hat{v}^2_{-k}}{n} + \frac{2}{n^2}\max\{\hat{v}^1_{-k} - \hat{v}^2_{-k},\ \hat{v}^2_{-k} - \hat{v}^3_{-k}\},\ \textit{and}$$

$$T_i(\hat{v}) = \frac{\hat{v}^2_{-i}}{n} + \frac{2}{n^2}\max\{\hat{v}^1_{-i} - \hat{v}^2_{-i},\ \hat{v}^2_{-i} - \hat{v}^3_{-i}\},\quad \forall i \in I \setminus \{k\}$$

The winner pays the second highest bid and then each agent is paid $1/n$ times the second highest bid amongst *other* agents plus $2/n^2$ times the larger of either the gap between the first and second highest other bid or the second and third highest other bid. The total subsidy (the negation of the revenue) under NR is:

$$\frac{2}{n^2}\sum_{i \in I}\max\{\hat{v}^1_{-i} - \hat{v}^2_{-i}, \hat{v}^2_{-i} - \hat{v}^3_{-i}\} - \frac{2}{n}(\hat{v}^2 - \hat{v}^3), \qquad (1)$$

which clearly goes to 0 as population size (n) grows.

Theorem 2. *In single-item allocation settings, the NR mechanism is asymptotically strongly budget-balanced as the population size grows, for arbitrary valuations with a finite bound.*

If Eq. (1) is a little opaque, it is easy to see that from it one can derive the crude worst-case bound of $\frac{2}{n}(\hat{v}^1 - \hat{v}^4)$: the required subsidy will never be greater than $2/n$ times the difference between the first and fourth highest bids.

To get a better sense of the workings of the mechanism, we turn now to the following 4-agent example:

v_1	v_2	v_3	v_4
10	8	6	4

Because both RM and NR are strategyproof, under either mechanism rational agents will announce their true values. Under RM agent 1 makes payment $8 - \frac{6}{4} = 6.5$, agent 2 receives payment $\frac{6}{4}$, and agents 3 and 4 receive payment 2; the resulting revenue is $8-1.5-1.5-2-2 = 1$. Under NR, we have additional redistribution to agent 1 of $\frac{2}{n^2}\max\{\hat{v}^1_{-1} - \hat{v}^2_{-1}, \hat{v}^2_{-1} - \hat{v}^3_{-1}\} = \frac{1}{8}\max\{2,2\} = \frac{1}{4}$, and $\frac{1}{2}$, $\frac{1}{2}$, and $\frac{1}{4}$ for agents 2, 3, and 4, respectively. The resulting utilities (and revenue to the center) are: $u_1 = 10 - 8 + \frac{6}{4} + \frac{1}{4} = \frac{15}{4}$, $u_2 = 0 - 0 + \frac{6}{4} + \frac{2}{4} = \frac{8}{4}$, $u_3 = \frac{10}{4}$, $u_4 = \frac{9}{4}$, revenue $= -0.5$.

In this example the subsidy required under NR is less than the revenue generated by RM—we are *closer* to strong budget-balance, which is a good thing—but this will not always be the case. For instance, if we modify the example such that agent 3's value is 7 rather than 6, RM will yield revenue 0.5 while NR will require a subsidy of 1. So neither mechanism comes closer than the other to perfect budget-balance *in general*. Thus a numerical analysis considering a distribution of valuations will be illuminating.

Figure 1 presents the expected revenue for RM and expected subsidy for NR—as a percentage of social welfare—as a function of population size for uniformly distributed values. Distance from strong budget-balance is small for both RM and NR for all population sizes, and it gets *very* close to 0 as population size grows from 3 to 10.

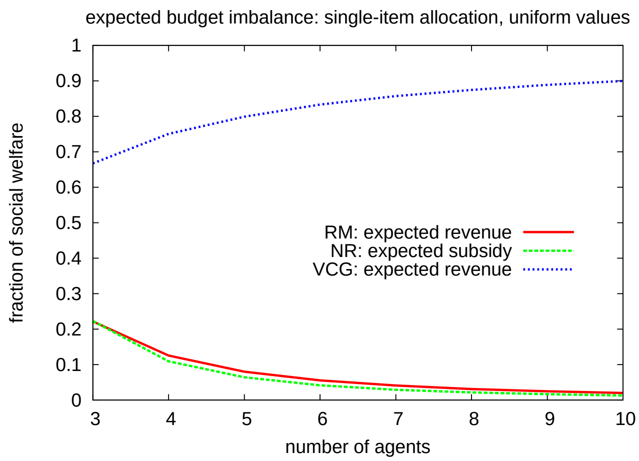

Figure 1: Expected distance from perfect budget-balance for a single-item allocation domain with values drawn i.i.d. from a uniform distribution.

3. CONCLUSION

This paper introduces a redistribution-style mechanism (the "no-revenue mechanism" NR) that is dominant strategy efficient, ex post individual rational, and *no-revenue*, requiring subsidies that are demonstrably quite small in single-item allocation settings; it can be viewed as a kind of transformation of the redistribution mechanism RM proposed by Cavallo [1]. In the full version of the paper we show how this transformation technique can be generalized to convert any efficient, IR, and no-deficit mechanism into one that is efficient, IR, and no-revenue. We also provide a demonstration that NR is effective in combinatorial allocation problems, and provide an analysis of the computational issues that arise as we move away from restricted domains such as single-item allocation. A detailed empirical analysis is also done. There are many compelling directions for future work, including design of efficient methods for computing redistribution payments, worst-case optimization, and leveraging of domain structure beyond that of allocation problems.

4. REFERENCES

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